ATTENUATION CHARACTERISTICS OF VLF WAVES FROM THE WAVEFORMS OF ATMOSPHERICS

MANORANJAN RAO

BOSE INSTITUTE, CALCUTTA, INDIA.

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ABSTRACT. In this paper a reevaluation of the data published earlier by Khastgir *et al* (1960) and Bhattacharya *et al* (1964) has been made to obtain the attenuation coefficients of VLF radio waves. The wave-guide mode-equation deduced by Wait (1957, 1958) has been used and the results obtained are compared with those of Taylor and Lange (1958). Attempt is made to explain the discrepancies between the results of the different authors.

INTRODUCTION

Ever since it had been established that lightning discharges radiated electromagnetic energy mainly in the VLF-band of frequencies, increasing attention of the various investigators in this field has been directed to the utilization of the naturally occuring VLF emissions for the study of the propagation characteristics of frequencies in the VLF-band. The main idea was to study how the spectra radiated by the lightning discharges varied with distance. Different workers adopted different techniques in recording and analysing the atmospherics or their waveforms for obtaining maximum information about the VLF-propagation with minimum possible assumptions. For example, Gardner (1950) recorded only the integrated levels of atmospheric noise on some chosen frequencies and studied how these levels changed with the time of the day. Bowe (1951), on the other hand, recorded tuned amplifier responses to individual sferics on certain chosen frequencies and determined how the radiation field on any frequency with respect to that on some reference frequency varied with distance. Similarly Chapman *et al* (1953), Taylor *et al* (1958), Taylor (1960), Hepburn (1959, 1960) Obayashi *et al.* (1959, 1956), and Croom (1964) also studied the change in the radiated spectra of atmospherics with distance and obtained reasonably accurate values for the attenuation coefficients for the VLF-frequencies. Khastgir *et al* (1960) and Bhattacharyya *et al* (1964) also made similar studies and following the method given by Bowe (1951) deduced how the ratio of field components on two frequencies varied with distance. In present paper an attempt is made to obtain the absolute values of the attenuation coefficients from the data published by Khastgir *et al* (1960) and Bhattacharya *et al* (1964).

THE METHOD OF ANALYSIS

While Khastgir *et al* (1960) recorded the tuned amplifier responses on some selected frequencies only (usually four in number), Bhattacharya *et al* (1964) subjected the recorded waveform to Fourier transformation by laborious numerical integration and obtained more or less continuous spectra within the range of 3-15 kc/s. These authors adopted the method given by Bowe (1951) which can be briefly summarised as follows :

The radiation field on any frequency f at a distance d can be written as

$$E(f, d) = \frac{A(f)}{d} \cdot \rho(f, d) \tag{1}$$

where $A(f)$ describes the source spectrum and $\rho(f, d)$ represents the attenuation suffered by the component frequency f and is a function of distance d. Obviously the distance occurring in the denominator accounts for the free-space decrement. If the amplitude at any frequeny f_n is measured relative to that at some reference frequency f_0 we can write,

$$\frac{E(f_n, d)}{E(f_0, d)} = \frac{A(f_n)}{A(f_0)} \cdot \frac{\rho(f_n, d)}{\rho(f_0, d)} \quad \ldots \tag{2}$$

Now if we assume that $A(f_n)/A(f_0)$ is statistically invariant, and if measurements are made on several lightning sources at different distances, then the ratio $E(f_n, d)/E(f_0, d)$ when plotted against d would indicate how $\rho(f_n, d)/\rho(f_0, d)$ behaves as a function of d. Following this argument of Bowe (1951) Khastgir *et al* (1960) and Bhattacharya *et al* (1964) gave plots of $E(f_n, d)/E(f_0, d)$ against d, for several values of f_n.

It has now been established that the VLF-waves when propagated to great distances over earth behave as if they are propagated through the waveguide formed by the lower boundary of the ionosphere and the earth. Extensive theoretical work has been done by several workers on this aspect of the problem viz. Budden (1961), Wait (1962). The expression deduced by Wait (1957, 1958) has been used by Taylor *et al* (1958) and Croom (1964) with consistent results. The expression deduced by Wait is given by :

$$E(f, d) = \frac{A(f)}{(a \ \text{in}\ d/a)^{\frac{1}{2}}} \cdot \exp\left[-\alpha(f)d\right] \quad \ldots \tag{3}$$

where $E(f, d)$ and $A(f)$ have the same meaning as in (1) and $\alpha(f)$ is the attenuation coefficient in nepers and a is the radius of the earth. In view of (3), the left hand side of (2) can be written as

$$\frac{E(f_n, d)}{E(f_0, d)} = \frac{A(f_n)}{A(f_0)} \exp\,[\alpha(f_0)-\alpha(f_n)]d \tag{4}$$

Taking logarithms we write :

$$\ln \frac{E(f_n, d)}{E(f_0, d)} = \ln \frac{A(f_n)}{A(f_0)} + [\alpha(f_0) - \alpha(f_n)]d. \tag{5}$$

Thus if the assumption is made that $A(f_n)/A(f_0)$ is a statistical constant of the source, the plots of $\ln \frac{E(f_n, d)}{E(f_0, d)}$ against d would give a straight line with a slope equal to the relative attenuation coefficient $[\alpha(f_0)-\alpha(f_n)]$.

RESULTS AND DISCUSSION

The data given by Bhattacharya *et al* (1964) for the ratio $E(f_n, d)/E(f_0, d)$ for different distances are replotted satisfying the equation (5) in fig. 1. The

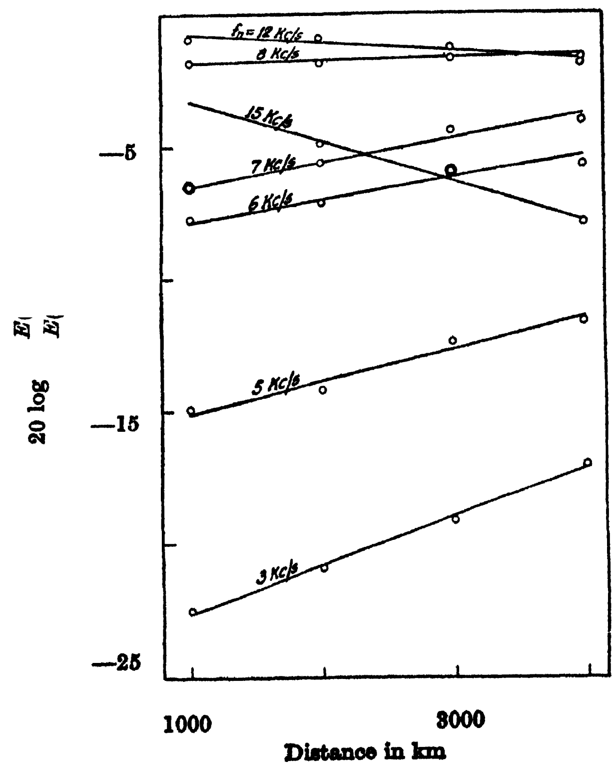

Fig. 1. Variation of 20 log $\frac{E(f_n,d)}{E(f_0,d)}$ with distance d for frequencies f_n = 3,5,6,7,8, 12 and 15 kc/s. (The reference frequency f_0 = 10 kc/s.)

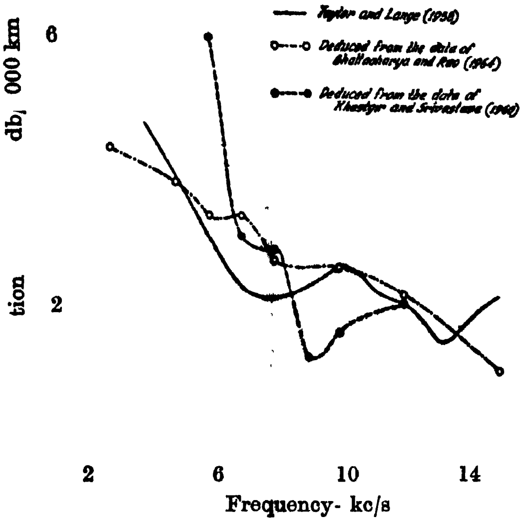

Fig. 2. Attenuation Coefficients in db/1000 km for frequencies in the range 3-15 kc/s —Comparison with the results of other workers.

ordinates are expressed in *db* and straight line approximation holds good at almost all the frequencies of 3, 5, 6, 7, 8, 12 and 15 kc/s. The reference frequency chosen by the authors was 10 kc/s and so the slopes of the straight lines in the figure correspond to the attenuation coefficients at the particular frequencies relative to that at 10 kc/s.

A similar calculation has been done for the data of Khsatgir *et al* (1960), and the relative attenuation coefficients have been determined. It has been found in this case that the plots showing $E(f_n, d)/E(f_0, d)$ against d are too scattered to yield a straight line. This is due to the fact that the distances reported by Khastgir *et al* are rather small (the maximum distance being 1600,km) Since equation (3) takes into account only the dominant mode in the series representation of the field component, it is valid only when the distance from the source is very great. Nevertheless, mean straight lines are drawn through the scatter of points representing the data of Khastgir *et al* and the relative attenuation coefficients have been determined (these mean curves are not shown). The reference frequency chosen by Khastgir *et al* is 12 kc/s. In order to effect a comparison between the results thus obtained and those given by others, the values of α at 10 and 12 kc/s have been taken from Taylor *et al* (1958) and the relative attenuation coefficients have been converted into absolute units. The final results are given table 1 and are illustrated in fig. 2.

It can be seen from fig. 2 that the results of Bhattacharya *et al* show a better general agreement with those of Taylor *et al* than the results of Khastgir *et al.* The main reason for this is, as already mentioned, the relatively

Table 1

Attenuation coefficients of VLF-waves for frequencies 3-15 kc/s

f Frequency in kc/s.	Attenuation coefficient α in db/1000 Km.		
	Taylor and Lange*	Bhattacharya and Rao	Khastgir and Srivastava
3	×	4.35	×
4	4.75	×	×
5	3.88	3.85	×
6	3.05	3.35	6.00
7	2.25	3.35	3.00
8	2.10	2.65	2.80
9	2.25	×	1.20
10	2.55	(2.55)	1.60
11	2.25	×	×
12	2.00	2.15	(2.00)
13	1.50	×	×
14	1.75	×	×
15	2.10	1.00	×

* These values are obtained approximately from the curves given by Taylor and Lange. The figures in brackets are taken from the data of Taylor and Lange for comparison (see text).

smaller distances of the lightning sources in the observations of Khastgir *et al.* Even between the curves corresponding to the data of Bhattacharya *et al* and to those of Taylor *et al* the discrepancies are quite apparent. The main source for these discrepancies is the assumption made by Bhattacharya *et al* regarding the constancy of spectral content of the source. Taylor *et al* (1958) overcame this unrealistic assumption by recording the same sferic at more than one station and by taking the ratio of the spectra at these stations. Also, while the data of Bhattacharya *et al* represent averages over a large number of sferics irrespective of the direction of travel, the terrain, etc, those of Taylor *et al* correspond to a few sferics originating from a particular storm.

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